Structural degradation models: an overview

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Abstract

Structural reliability is commonly approached by evaluating observations of system failures over time. Although this approach has been very useful, it leaves out the insights of the process leading to failure. The understanding of this process, which is referred to as deterioration, plays a significant role in life-cycle analysis. Degradation depends on the type of structure and the nature of the external or internal demands (i.e., frequency, intensity, mechanism). Therefore, there are important challenges in managing the uncertainty of this process. This paper presents an overview of the problem of degradation and the associated uncertainties; and suggests different modelling alternatives.

Keywords: Life-cycle analysis, degradation models.

1 Introduction

Studying and modelling degradation of infrastructure systems is essential for life-cycle analysis, structural reliability estimations, and for defining future investments in maintenance and operation. Modelling degradation requires understanding of deterioration mechanisms and the process of damage accumulation, as well as, finding alternatives to handle the associated uncertainties. This means the assessment of transient states (time-dependence change) of any performance indicator (or a structural property) as a result of the dynamic interaction of the structure with external (environmental) factors. Although several structural deterioration mechanisms have been studied there is still important challenges on the modelling side and on the strategies to incorporate them within the life-cycle analysis. This paper presents a description and a critical review of the problem of degradation and suggests some tasks for the future.

2 Life-cycle analysis

A central element in modern engineering is the concept of life-cycle analysis, which studies the performance of a system over a finite or infinite time horizon. Life-cycle analysis has been used for different purposes among which the use as a project evaluation tool is maybe the most common. Within this context, it is called *life-cycle cost analysis* (LCCA). The LCCA is based on the evaluation of the following cost-benefit objective function:

$$Z(\mathbf{p}) = B(\mathbf{p}) - C_0(\mathbf{p}) - \sum_{i=0}^{n} D_i(\mathbf{p}) \tag{1}$$

where $B(\mathbf{p})$ are the benefits derived from the existence of the structure, $C_0(\mathbf{p})$ is the initial investment (e.g., design and construction costs) and $D_i(\mathbf{p})$ are the costs in the which the owner may incur as a result of the operation (e.g., maintenance and reconstruction after failure, payment of insurance policies, etc.). Note that all these costs are evaluated at discrete points in time and most of them are uncertain.

According to decision theory, the function $Z(\mathbf{p})$ should be evaluated in terms of expected discounted costs (i.e., net present value of discounted costs). Thus, by including the time horizon of the analysis, T, the best decision requires solving the following equation,

$$\mathbb{E}[Z(\mathbf{p},t)] = \mathbb{E}\left[B(\mathbf{p},t)h(\gamma,t) - C_0(\mathbf{p} - \sum_{i=1}^{n} D_i(\mathbf{p},t_i)h(\gamma,t_i)\right] \tag{2}$$

where $h(\gamma)$ is the discount factor, which is a function of the discount rate γ that may or may not be dependent of time. Note also that this evaluation can be carried out for finite time horizon t or for an infinite time span, $t = \infty$.

Clearly, for the problem presented in equation (2), the project is feasible in the region where $E[Z(p)] > 0$. Furthermore, it should be point out that the LCCA has the ability to integrate within a single framework both the mechanical performance of the system and the associated cost of investments throughout its lifetime; for instance, investments in maintenance and repair after failure. Decisions about future investments are directly related to the performance of the system over time, which is clearly dominated by degradation mechanisms. In this paper we will focus on specific stochastic models that describe the system performance as a result of any physical phenomena (e.g., corrosion, creep, fatigue, earthquake damage).

3 Structural performance over time

3.1 Definition of degradation

Deterioration (degradation) is defined as the loss of value of a structural property (e.g., stiffness) or the detriment of any performance indicator that describes the structure's overall behaviour (e.g., drift, reliability index β). When modelling degradation there are two main problems: understanding the physical nature of degradation and modelling its stochastic nature.

Degradation is in essence an unobservable entity; and only its manifestation is an observable and frequently measurable quantity (Ben-Akiva and Ramaswamy, 1993). In other words, deterioration cannot be measured directly, whereas its effects do result in measurable damage. This means that the consequences of deterioration that can be observed (e.g., cracks) are often mistakenly used as descriptors of the deterioration (damage) process. Observations of damage such as cracks or structural deformations are often only indicators for the *latent* condition of the system. Thus, modelling degradation is actually about those intrinsic variables and processes that eventually manifest in the form of damage (deterioration). Because the process leading to damage is unobservable, most engineering degradation models relate an index, which describe the change of the system's condition with time, with a set of explanatory variables. Generally these indices use linguistic descriptions (e.g., "poor", "moderate", "excellent"); also arbitrary numerical scales or specific performance measures (e.g., safety measures) that represent the system's condition may be used. Despite the importance of addressing the actual nature of degradation, this topic will not be discussed in this paper; the main emphasis will be on modelling the stochastic nature of the process.

3.2 System evolution over time

Structural systems are designed and built to operate under acceptable conditions over a pre-specified time window. However, both internal and environmental factors affect its performance causing that the system does not achieve its objective during what is usually referred to as the system's time-mission. The deviation of the structural performance from the original design conditions is called degradation. Thus, the system starts operating at an initial condition u_0 (performance measure, e.g., capacity), which is an uncertain quantity, and due to deterioration this condition declines with time. This capacity reduction is frequently evaluated as combination of sudden and slow progressive variations of the system condition. An illustrative sample path of the degradation process of a system is shown in Figure 1. Once in operation, if the system condition falls bellow a predefined minimum (acceptable) performance threshold (e.g., k^* or s^* in Figure 1), before its mission time, the system is intervened by carrying out either a preventive or corrective maintenance. Repair times are usually modelled as random variables since degradation is clearly an uncertain process; furthermore, in practice repairs times also depend on the system condition at the time of intervention and on the availability of resources. Intervention times are important in many mechanical systems; however, in large civil infrastructure, due to the fact that the mission time is frequently large (i.e., $t_m > 75$ years), it is frequently modelled as instantaneous positive changes in the system condition.

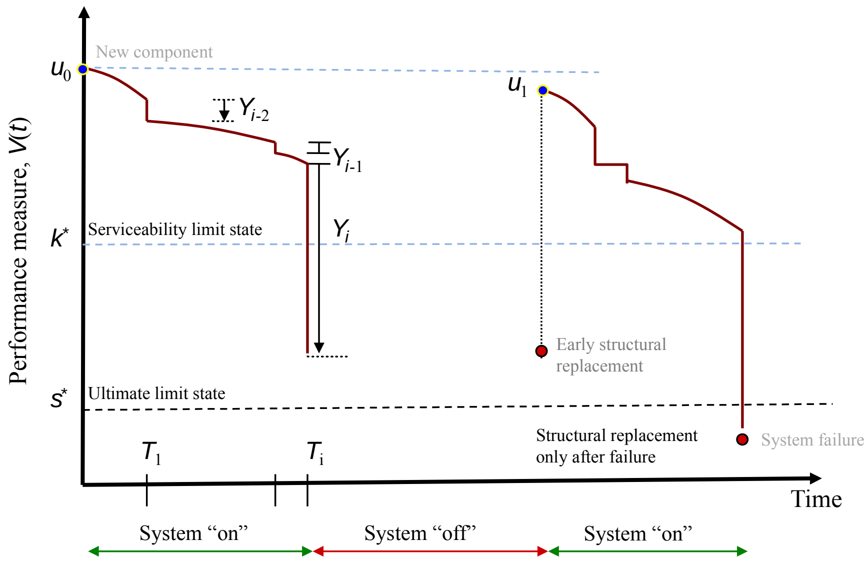

Figure 1. Illustration of structural performance over time.

Under the structural performance scenarios described above, decisions about investments should be carefully reviewed. These decisions include, among others, the selection of the design parameters and the inspection and maintenance policies. This decision process is commonly structured as a cost-based optimization problem, although other criteria such as utility or environmental criteria can be employed also.

4 Reliability quantities within the context of LCCA

Consider a system whose condition (performance indicator) at any time t is $V(t)$. Furthermore, let's define the structure's lifetime, L, as a random variable describing the length of time required for the structure to reach a predefined performance threshold; e.g., k^*, with $k^* \leq u_0$; where $V(t) = u_0$ is the state of the system at time $t = 0$. In practice, k^* is defined as a limit state (e.g., serviceability, ultimate). Therefore, if $D(t)$ is the total degradation at a given time, the performance level at time t can be computed as $V(t) = \max(u_0 - D(t), k^*)$ and the structural lifetime can be written as:

$$L = \inf\{t \geq 0 : V(t) = k^*\}. \quad (3)$$

Consequently, the structure's reliability can be expressed as:

$$R(t) = P(L > t), \quad (4)$$

where t is the time at which the system is evaluated. In many cases, and in life-cycle analysis in particular, a quantity of interest is $t = t_m$, where t_m is the length of the system's *mission*; i.e., the lifetime for which the system is intended. Note that computing the lifetime can be interpreted also as a first passage probability problem.

Reliability can be expressed also in terms of a performance indicator as:

$$R(t) = P(L > t) = P(V(t) > k^*) = P(u_0 - k^* - D(t) > 0). \quad (5)$$

5 Modelling degradation

5.1 Separation of the degradation mechanisms

It can be noticed from equation (3) that a central element in reliability analysis is the appropriate evaluation of degradation, i.e., $D(t)$, which can be expressed in a general form as:

$$D(t) = \int_0^t \delta(p, \tau) d\tau \tag{6}$$

where $\mathbf{p}$ is a vector parameter that define the structural performance and δ is the degradation function (as a result of all causes). Clearly the complete definition of δ in equation (6) requires the identification and understanding of all degradation mechanisms and their interaction, which is not possible. Therefore, in practice, degradation is usually restricted to a small set of mechanisms. Thus, the following approximation is frequently used:

$$D(t) = \int_0^t \delta(\mathbf{p}, \tau) d\tau \approx \sum_{i=1}^{m} \int_0^t \kappa_i(\mathbf{p}, \tau) d\tau = \sum_{i=1}^{m} d_i(\mathbf{p}, t) \tag{7}$$

where $d_i(p,t)$ is the degradation function that describes the mechanism i. The two most common degradation mechanisms are progressive and shock based degradation (Sanchez-Silva et al., 2011), which are described in the following subsections.

5.2 Progressive degradation

In reinforced concrete structures, progressive deterioration is caused by phenomena such as chloride ingress in RC, which leads to steel corrosion, loss of effective cross-section of steel reinforcement, concrete cracking, loss of bond and spalling. The details of these deterioration mechanisms are beyond the scope of the paper but are well described by, for instance, Mori and Ellingwood (1994), Duracrete (2000), Val and Stewart (2005), Bastidas et al. (2009), Biondini and Frangopol (2009). Besides, the effect of corrosion on steel structures has been extensively studied by Melchers (1999, 2003 and 2005) and Shi and Mahadevan, (2001). Finally, the combined effect of corrosion and fatigue has also been studied by Zhang and Mahadevan, (2001) and others.

Most progressive degradation models available in the literature assume that the form of the degradation process is known, but the parameters are uncertain. The solution to this problem conveys to a parameter estimation problem. Thus, if $V(t)$ is the state of the system at a given time t; which in practice, may be expressed in terms of, for example, remaining capacity, reliability, safety, durability, etc; then, these type of models have the following general form:

$$V_p(t) = \begin{cases} u_0, & \text{for } 0 \leq t \leq t_e \\ u_0 - h(\mathbf{p}, t - t_e), & \text{for } t > t_e \end{cases} \tag{8}$$

where u_0 is the remaining life of the system at time $t = 0$. The function h may take a linear, non-linear, exponential of any other form based on the appropriate selection of the vector parameter $\mathbf{p}$, which depends upon the problem at hand.

Progressive degradation can be handled also in terms of the degradation rate, i.e., $\delta_p(\mathbf{p}, t)$, $t \geq 0$. Thus, the system state (e.g., capacity measured in physical units per unit time) at time t can be expressed as:

$$V_p(t) = \int_0^t \delta_p(\mathbf{p}, \tau) d\tau \tag{9}$$

where, the rate δ_p may be dependent or independent of time.

The uncertain nature of the process can be taken into account also by making the vector parameter $\mathbf{p}$ in $\delta_p(\mathbf{p}, t)$ a random variable; these are usually called random variable (RV) models. However, this model does not account for the temporal variability of the process as pointed out by Pandey et al. (2009). As an alternative, the temporal variability of the phenomenon can be modelled by defining the parameters (or the performance/deterioration itself) as a *stochastic process* (Pandey et al., 2009). Within these models, gamma processes (GP) and Gaussian processes (e.g, Brownian motion) are the best option (Abdel-Hameed 1975, van Noortwijk 2009).

Table I shows a comparison between the progressive degradation models in terms of temporal variability (randomness), expected deterioration $\mathrm{E}[D(t)]$ at any time t, and the methods (analytical, numerical or by simulations) for the reliability.

5.3 Shock based degradation

Shocks can be defined as events that cause a significant change in a system's performance indicator (e.g., physical property) in a small time interval. Then, shock-based degradation occurs when a fixed amount of capacity/resistance is removed from the system at discrete points in time (Sanchez-Silva et al. 2011). Shock-based deterioration is frequently caused by extreme events such as earthquakes, hurricanes or blasts (including both accidents and terrorists attacks). Extensive research has been carried out on mathematical models for shock degradation; for more details see Barlow and Proshan (1965), Aven and Jansen (1999), Nakagawa (2005), Feldman (1977). Among the first works on this topic in civil engineering were published by Rosenblueth E, and Mendoza E. (1971) and Rosenblueth E. (1976) within the context of earthquake resistant design optimization. Their ideas were later reconsidered by Rackwitz (2000) to propose a general framework for optimal design and reliability verification. Recently, Sanchez-Silva et al. (2011) proposed a model for damage accumulation as a result of extreme events.

Table I: Features and comparison of the typical models for progressive degradation.

Model	**Temporal variability**	**Deterioration trend** E[$D(t)$]	**Reliability analysis**	**Computational cost (Exec. times)**
Deterministic	No	General	-----	Low
RV	No	General	Monte Carlo, analytical and numerical	Medium / Low
Gamma Process ($c(t)$, β)	Yes	General: $c(t)$ / β	Analytical and numerical	Medium / High
Shock - approximation	Yes	General	Numerical and Monte Carlo	High

Let's define the random variable Y as the shock size (sudden damaging event). Then, if all shocks are *iid*, the total degradation at a given time t is given by:

$$D(t) = \sum_{i=1}^{N_t} Y_i \tag{10}$$

where N_t is a random variable describing the number of shocks that occur by time t. Furthermore, if L is a random variable that describes the life of the structure, the structural reliability can be computed as $R(t) = P(L > t) = P(D(t) < (u_0 - k^*))$; where; u_0 is the state of the system at time $t = 0$. Then, for shock-based degradation, the structural reliability at time t can be computed as

$$R(t) = \sum_{k=1}^{\infty} \bar{P}_k(u_0 - k^*)P(N_t = k) \tag{11}$$

The term $\bar{P}_k(u_0 - k^*)$ describes the probability that the system survives k shocks and $P(N_t = k)$ is the probability that there are k shocks by time t.

There are several models for modelling shock-based degradation. First, if the inter-arrival times are supposed independent and identically distributed (*iid*) following an exponential distribution with rate μ, and the shock sizes are also *iid* following a general distribution, degradation can be modelled as compound Poisson process (CPP). If the exponential rate of shock occurrence times is time-dependent ($\mu(t)$), the best model is a non-homogeneous compound Poisson process (NH-CPP), which is more general than the CPP in the sense that can model non-linear deterioration trends. An approach that helps with some complex computational aspects of damage accumulation –i.e., evaluation of convolutions- is the use of the so-called Phase-type (PH) distributions (Riascos-Ochoa et al., 2014). This special kind of distributions can be fitted to any positive distribution (or dataset) because they are dense in the set of positive distributions. Moreover, the reliability estimation is easy-to-evaluate because of their matrix-geometric properties. Also, they can model general deterioration trends. In Table II there is a comparison among various models in terms of temporal variability, deterioration trends, and the methods for the reliability estimation.

Table II: Features and comparison of the typical models for shock-based degradation.

Model	**Temporal variability**	**Deterioration Trends** E[D(t)]	**Reliability analysis**	**Computational cost (Exec. times)**
General Ti and Yi	Yes	General	Monte Carlo, Convolution integrals (numeric)	High
CPP $Ti \sim$ EXP(μ) $Yi \sim$ Gen.	Yes	Linear: μ E[Y] t	Monte Carlo, Convolution integrals (numeric)	High/Medium
NH - CPP $Ti \sim$ EXP($\mu(t)$) $Yi \sim$ Gen.	Yes	General: $\mu(t)$ E[Y]	Monte Carlo, Convolution integrals (numeric)	High/Medium
PH – Type (General) $Ti \sim$ PH and $Yi \sim$ PH	Yes	General	Matrix analytic (numeric)	Medium

6 Challenges of degradation modelling

Some of the main challenges for modelling degradation are:

1. fitting data to a particular degradation model
2. definition of the initial (as-built) condition of the structure, i.e, U;
3. definition of the times between shocks, i.e., T_i and F_T;
4. definition of shock-size distributions, i.e., F_Y;
5. damage accumulation process, i.e., $D(t)$.
6. combination of degradation mechanisms.

The first challenge is that degradation models should be constructed from field data. This is not an easy task in most large infrastructure projects mostly because in many cases degradation is a very slow process in time and, therefore, information is not usually available. In cases where information is available, the main problem is to fit the correct model to the data. For example, in Figure 2, it is shown that both progressive and a jump models fit the data rather acceptably, but they represent completely different damage mechanisms. Although this problem can be partially solved by studying the physical phenomena, the usual coupled effect of damaging mechanisms is difficult to capture. Finally, even if the trend or mechanism is identified, fitting the model parameters maybe also a difficult task.

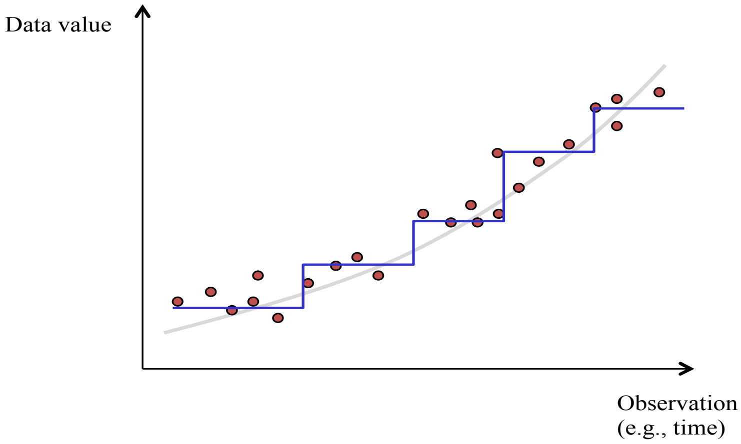

Figure 2. Fitting degradation data to both progressive and shock-based models.

The second challenge is to define accurately the initial condition of the structure; i.e., $U = u_0$. Although in practice structural elements are designed for specific moment and shear forces, the actual capacity after construction (as-built condition) is difficult to estimate accurately. There is uncertainty in the design models and parameters, in the material properties and in the construction processes. These uncertainties might have high coefficients of variation. There are not many studies that address this issue and the basic assumption is that initial performance is deterministic and defined as the performance level specified in design. This value is important when evaluating degradation since it defines the structural capacity available; i.e., u_0 - k^*.

For the particular case of shocks, the third challenge is related with the definition of times at which shocks occur. These times are closely related to the events that cause the shock. Some important cases include problems where damage is caused by natural events (e.g., earthquakes, hurricanes); in these cases, the NH-CPP model (Table II) is commonly applied. Consider the general case where the times between events are independent random variables T_i, with probability distribution F. Then, the time until the k-th shock is

$$S_k = \sum_{i=1}^{k} T_i \qquad \text{with distribution:} \qquad F_{S_k}(t) = P\left\{\sum_{i=1}^{k} T_i \leq t\right\} \tag{12}$$

consequently, the probability to have k shocks by time t, i.e., N_t, is also a random variable with distribution:

$$P\{N_t = k\} = P\left\{\sum\nolimits_{i=1}^{k} T_i < t\right\} - P\left\{\sum\nolimits_{i=1}^{k+1} T_i < t\right\} = F_{S_k}(t) - F_{S_{k+1}}(t) \tag{13}$$

Clearly this expression requires computing a convolution for which analytical solutions can only found in few specific cases.

The fourth problem is the definition of shock size distribution, i.e., F_Y. Contrary to shock time occurrences, shocks sizes does not refer to the external event that causes a sudden decay in the system performance. Shocks are evaluated in terms of the structure's performance indicator and, therefore, they depend of the relationship between the external event (e.g., hazard) and the structural characteristics. For example a shock is not an earthquake event, but the consequences (damage) as a result of the earthquake occurrence. Although it is not easy to find the shock size distribution, in many cases it is possible to make generalizations based on observations; for instance, structures constructed with the same materials and with similar topological characteristics.

The other important source of uncertainty is the damage accumulation process, which requires computing the loss of capacity after a given number of shocks *k*; i.e,

$$D_k = \sum_{i=1}^{k} Y_i \tag{14}$$

Most models work on the assumption that shocks are independent and identically distributed; therefore, the probability distribution of the structural performance level (state) after *k* shocks is:

$$P_k(v) = P(D_k \leq v) = P\left\{\sum_{i=1}^{k} Y_i \leq v\right\} \tag{15}$$

Frequently the structure's failure is defined as the case in which the performance indicator falls bellow a pre-specified threshold k^* (limit state). Then, probability of surviving after *k* shocks can be computed as:

$$\bar{P}_k = P\left\{\sum_{i=1}^{k} Y_i \leq u_0 - k^*\right\} = P\{D_k \leq u_0 - k^*\} = F_{D_k}(u_0 - k^*) \tag{16}$$

which again requires computing a convolution. Furthermore, an important element of damage accumulation is the assumption that shocks sizes are identically distributed. This is clearly not the case in many real problems and, although in many cases this assumption can be ignored, it may lead to incorrect assessments of probability. For instance, consider a structure located in a seismic region subject to a series of earthquakes that cause damage to accumulate with time. Then, the damage caused by a given earthquake is conditioned on the damage state of the structure just before its occurrence. This means that the probability distribution of next shock size (i.e., damage) is somehow dependent on the current damage state (i.e., history of shocks).

There are two basic approaches to modelling the increasing nature of damage accumulation with time: 1) defining a function of shock size distributions; and 2) conditioning on the system state (Junca and Sánchez-Silva, 2013). The first approach consists in finding a function of shock size distributions that keeps track of accumulated damage (Figure 3).

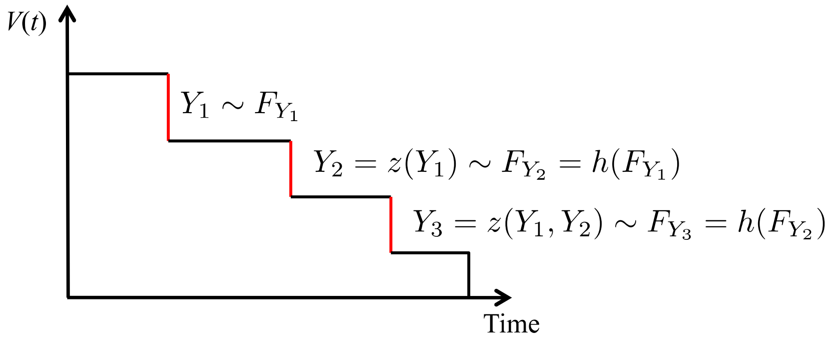

Figure 3. Increasing damage accumulation process.

Consider the sequence of shocks Y_i, where i=1,2,... indicate the order of the arrivals. Then, it is reasonably to assume that there exists a relationship between shock distributions as follows:

$$F_{Y_{i+1}} = h(F_{Y_i}) \tag{17}$$

where h is a positive continuous increasing function. The selection of function h should be made carefully to keep some important stochastic properties of the process. Two convenient models that follow this structure are the *Geometric* and *Gamma* processes. In the Geometric process, it is assumed that for a sequence of independent and nonnegative random variables Y_i, i =1,2,... with common probability distribution F, the distribution function of Y_i is given by $F(a^{i-1}y)$; where a is called the ratio of the process (Lam, 2007). On the other hand, the gamma process handles this problem by making the parameters of the gamma distribution time-dependent (Noortwijk, 2009).

The second approach is to define a function $g(y,v)$, which should be continuous, non-decreasing in y (shock size) and non-increasing in v (structural performance indicator). Thus, this model assumes that shock sizes, i.e., Y_i, are *iid* and that shocks occur at times T_1, T_2,… The degradation caused by shock Y_i is defined in terms of g(Y_i, $V(T_i^-)$). Then, the state of the system at a given time t can be computed as:

$$V(t) = z - D(t) = z - \sum_{i=1}^{N(t)} g(Y_i, V(T_i-)). \tag{18}$$

An example of function g could be $g(y,v) = \beta(y/v)$; where β is a constant, y is the shock size and v is the structural condition.

Extensive research has been carried out on developing mathematical models for shock degradation (Barlow and Proschan (1965), Aven and Jensen (1999)). Although this problem has been discussed extensively in civil engineering related problems, few analytical solutions have been proposed within the context of structural optimization and life-cycle cost analysis (Table II).

Finally, the problem of combination of degradation mechanisms (progressive and shock-based) is of great importance. In many deteriorating systems both types of degradation mechanisms are present. For instance, the effect of corrosion (progressive degradation) and sudden events as earthquakes (shock-based degradation) in infrastructures is of particular importance. In Sanchez-Silva et al. (2011) a mathematical framework that includes both types of degradation models is proposed as a general shock-model and a deterministic function for the progressive degradation. However, the mathematical expressions obtained for the reliability quantities cannot be solved numerically in an easy way. In Klutke et al. (2002) a more tractable expression for the mean of the lifetime is obtained, but considering only a linear deterministic model for progressive deterioration and a shock-model as a CPP. Iervolino, et. al. (2013) work a more general case of deterioration as a combination of a gamma process and a shock model as a CPP with shock magnitudes distributed gamma (and exponential as an special case). The reliability quantities are also difficult to obtain numerically in this case. Then, a need still exists for proposing and solving general deterioration models by appropriate numerical methods in adequate mathematical terms. An alternative that has been addressed recently is to use *Lévy processes*, which are stochastic processes with stationary and independent increments. The amenability of this formulation is that it may allow the modelling of both types of degradation as an independent combination of processes.

7 Conclusions

Design and operation of infrastructure requires an integrated approach that takes into account actors (owner, contractor, user) and processes (design, construction, operation) and the mechanical performance, which is at the heart of the process. Inevitably, the environmental conditions, the demands and the natural aging process lead to the system's performance decay over time. Then, optimum solutions necessitate the selection of appropriate degradation models. Within the context of this work, the main challenges in modelling structural degradation can be summarized as follows:

- to find dependable mathematical models that accurately describe the data and the underlying physical processes associated with degradation;
- to characterize and validate the stochastic nature of the degradation process;
- to find either an analytical or numerical solution to the stochastic problem that can be reasonably easy to implement; and
- to validate mathematical models.

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