Optimal geometric configuration of the solder joint by using structural reliability-based optimization

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Abstract

The reliability is a major concern in the microelectronic industry. However, new packaging and assembly technologies and requirements to comply with environmental friendly directives create challenges in assuring the reliability of mechatronics products for higher performance and lower cost. The deterministic design modelling does not take into account the uncertainties of design parameters and the variability of operating conditions in service. In this work, the reliability-based design optimization is developed to search the optimal geometrical configuration of the solder that fatigue performances are better. Moreover, the RBDO formulation allows for dealing uncertainties related to geometrical dimensions and temperature loading. The RBDO approach is combined to the kriging approximation in order to surrogate the thermo-mechanical nonlinear finite element model. This paper demonstrates the effectiveness of the RBDO method with kriging metamodel to conduct the design optimization of complex structures.

Keywords:kriging metamodel, reliability, optimization, fatigue, solder joint

1. Introduction

Embedded electronic systems have a role increasingly growing up in transport: electric and hybrid vehicles, trains and aircraft. For these applications, security and reliability are a critical point. New packages are characterized by smaller sized or more integrated components, finer pitches, and new materials. New requirements to comply with environmental friendly directives create challenges in assuring the reliability of mechatronic products for higher reliability and lower cost. However, mechatronic packages operate in hostile environments including wide temperature ranges and vibrations in which multiple interactive damage mechanisms occur. Field returns show that most reported failures in automotive mechatronic devices are due to the fatigue failure of solder joints. In order to meet reliability requirements the robustness of solder connexions should be validated.

Due to thermal expansion and stiffness mismatch of the different materials of the device assembly, thermal cycles induce mechanical stresses in the solder joints connecting electronic components to the circuit board. When the device is operational and subjected to external environmental conditions, these stresses can be significant leading to plastic deformation due to the visco-plastic behaviour of the solder. Accumulated plastic strain can cause damage leading to crack initiation and to solder joint failure. Consequently, to avoid field failures and reduce warranty costs, robustness validation of critical solder joints should be done as early as possible in the development process of an automotive mechatronic device.

The reliability prediction of the mechatronic devices is usually estimated through the mean time between failures (MTBF). This calculation is based on the handbook methods (FIDES, MIL-HDBK-217F…etc) and on individual failure rates for each component constituting the mechatronic device which are statistically obtained from field data. It is well documented that this estimation can leads to poor prediction [1].

However, in order to predict the reliability of mechatronic devices, the failure mechanisms should be well understood, so to predict correctly the lifetime. The physical failure approach aims to modelling the failure mechanisms by combining mathematical modelling with accelerated life testing to predict the reliability of devices [2]. The finite element method (FEM) is generally used to modelling the physical failure mechanisms in solder joints subjected to temperature cycling to predict the solder joint fatigue life. These simulation tools are based on a deterministic approach which does not take into account the variability of input parameters and the uncertainties related to design, loading, and operational conditions. A more rigorous approach for predicting the solder joint fatigue lifetime and the reliability of mechatronic devices should take into account the uncertainties arising from the random nature of the temperature fluctuations caused by power transients and thermal environment changes, the thermal expansion mismatch of the different materials of the assembly, the material properties and the fabrication process.

Several authors have used the deterministic design optimization to find the optimal design of the solder joint and improving the reliability of the device [3-5]. The design of the solder joint is searched by minimizing or maximizing specific performance requirements. However, the deterministic design optimization (DDO) does not take into account the uncertainties and generally leads to unreliable design. For this reason, the Reliability-Based Design Optimization (RBDO) is developed to balance cost and safety [6]. The RBDO formulation involves the evaluation of probabilistic constraints performed by the reliability analysis, which can be done either by stochastic simulations or by moment methods. As Monte Carlo simulations are computationally expensive, the moment methods, especially the first order reliability method (FORM), are widely used in RBDO procedures, due to their simplicity and efficiency. However, RBDO problem remains a difficult task since it requires the coupling of optimization algorithms, reliability analysis and simulation tools (i.e, FEM). This situation becomes impracticable for complex structures when finite element analysis considering nonlinear material behaviour and fatigue life prediction analysis are involved. Several authors are developed different RBDO formulations to overcome the computational effort and the numerical difficulties, which are still the main barrier for practical engineering applications [7].

In this work, the RBDO of the solder joint of a mechatronics device is investigated. It aims to find the optimal geometrical configuration of the solder that thermo-mechanical performances of the solder joint are optimized for better fatigue life. The finite element method (FEM) is used for modelling the solder joint subjected to temperature cycling and to predict solder joint fatigue life. Due to time consumption of the numerical model, the kriging metamodeling approximation is adopted in order to surrogate the performance functions because it allows one to quantify the surrogate error [8]. In this work, the RBDO approach combined to the kriging metamodel updating is developed to perform the reliability-based design optimization problem.

The RBDO formulation used in this work consists to minimize the probability of failure under feasibility constraints. Thus, the RBDO problem is transformed to several cycles followed by updating the kriging approximation in the neighbourhood of the limit state function around the optimal point of the previous cycle. A selected sample points are added to the kriging approximation by using the technique of constraint Boundary sampling (CBS) proposed by Lee et al. [9] to enhance the efficiency of constructing kriging models. This iterative scheme with updating the kriging metamodel is repeated until convergence. The numerical results show the efficiency and the attractiveness of the proposed RBDO methodology for the mechatronic systems.

2. Finite element modelling

2.1 Solder joint finite element model

In order to predict reliability in service it is a good practice in the electronics industry to combine accelerated testing and a physics of failure approach and have a deeper understanding of the failure mechanisms. Experience shows that failures in automotive electronics assemblies are often caused by thermal fatigue damage. For example, when damage accumulation in a solder joint reaches a certain critical level, this leads to an electrical or a mechanical failure. The physics of failure approach requires models that predict the stress and strains in the packaging materials of the assembly when the device is operational and subjected to external environmental conditions. Finite Element Method (FEM) is often used to predict the mechanical solder joint degradation. This degradation evolves with repeated thermal and power cycling and ultimately causes solder joint cracking, fissure propagation and interconnect failure.

The electronic package under investigation is a 256-pin plastic quad flat package with 0.5 mm pitch gull-wing leads, which is assembled on a board fixed in a tight metallic housing. In application, this whole mechatronic device is mounted in a vehicle under the hood. Figure 1 shows a perspective view of the meshed 3D global model. The model is based on the geometry of the Printed Circuit Board (PCB), the moulding compound and its solder interconnects and gull-wing leads. Calculus of the global model for only one thermal cycle has a very high computational cost.

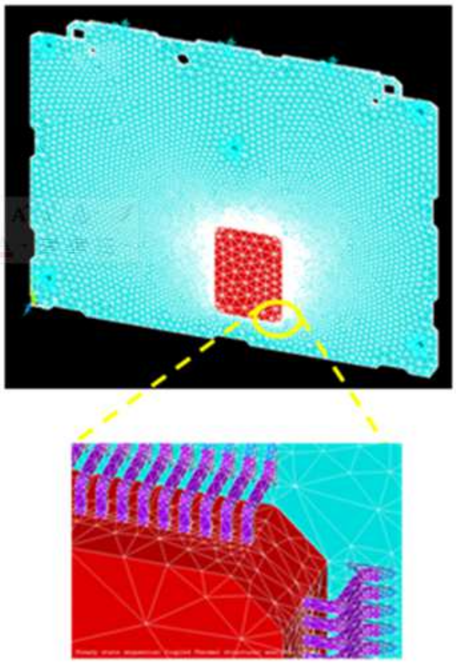

Figure 1. Global finite element model of the 256-PQFP.

In order to reduce the computational time, a local finite element model of the worst case solder joint is developed. This 2D plane strain model is based on the geometry of a single solder joint and different layer of materials, respectively the FR4, the copper lead and the moulding compound, as shown in Figure 2. The 2D finite element solder joint model allows us to model precisely the inelastic strain of the solder joint. It consists in describing the worst case solder joint with fine mesh elements and in taking into account material nonlinearities. The solder material is assumed to have a rate dependent plasticity. The solder joint finite element model is solved for five thermal cycles, where the first thermal cycle is illustrated in Figure 3.

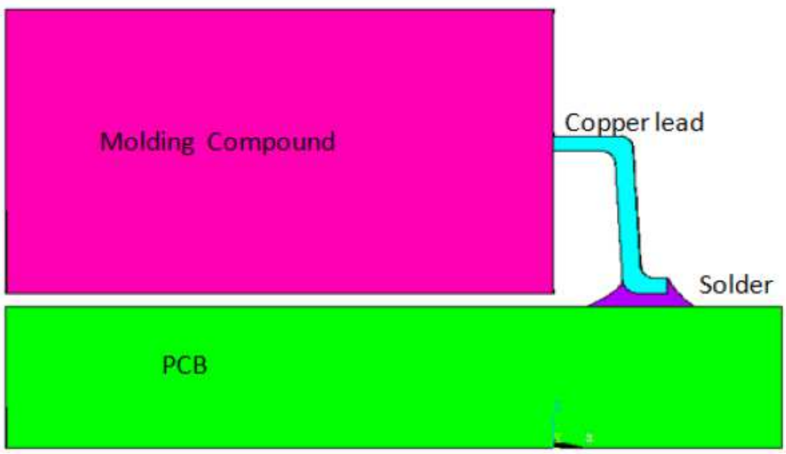

Figure 2. Geometry of the local solder joint model**.**

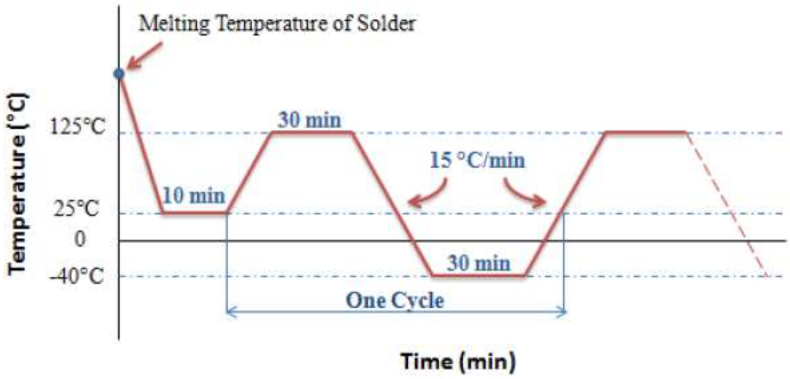

Figure 3. Cyclic temperature profile.

The commercial finite element analysis (FEA) software ANSYS V.13 was used for the nonlinear finite element computation. Figure 4 shows the finite element mesh of the solder joint model. As the temperature loading is time dependent, several substeps are required to carry out the nonlinear analysis and to ensure the convergence. For instance, the local model is as well time-consuming. However, the correctness of the finite element analysis is significantly dependent on the mesh density and the substeps iterations, accuracy of material properties used in the model and suitability of constitutive material models. Some of the assumptions made in the numerical model are made: all materials including the solder joint were assumed homogeneous; inter-metallic growth was not considered; linear elastic behaviour with temperature dependent properties was assumed for the package materials except the solder joint were is assumed visco-plastic-creep material behaviour.

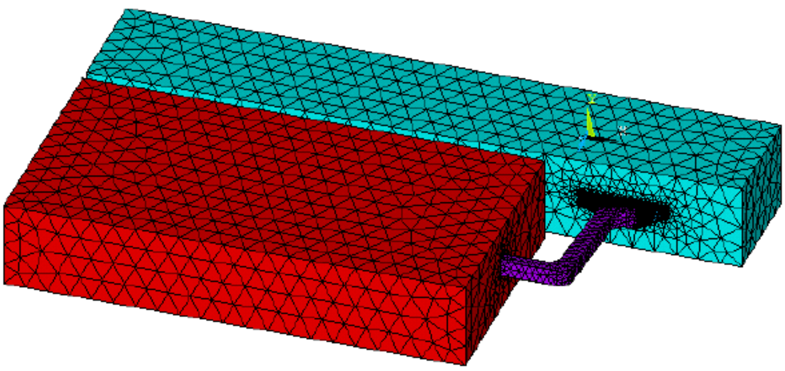

Figure 4. Finite element mesh of the solder joint model

2.2 Material properties and thermal loading

The thermo-mechanical behaviour of FR4 PCB board is assumed as orthotropic, linear elastic and temperature dependent properties, as given by [3]:

$$\begin{aligned} E_X(T) = E_Y(T) &= 27924 - 37T \text{ (MPa)} \\ E_Z(T) &= 12204 - 16T \text{ (MPa)} \end{aligned} \tag{1}$$

where T is in K°. The molding compound material is assumed to be isotropic and linear elastic. The copper lead is assumed isotropic linear elastic and temperature dependent [10] given by:

$$\begin{aligned} \alpha_{Copper}(T) &= 15.64 + 0.0041T \ \text{(ppm/ K)} \\ E_{Copper}(T) &= 141.92 - 0.0442T \ \text{(GPa)} \end{aligned} \tag{2}$$

where T is in . The solder material called SAC305 (Sn96.5Ag3Cu0.5) are assumed with elasto-plastic-creep behaviour, as creep plays a very important role in the deformation behaviour of solder joint at a homologous temperature [11]. All material properties shown in Table I are given for the reference temperature 20 C°. In solder joint materials, the development of plastic strains is dependent on the rate of loading. One of the equations developed is Anand's model which incorporates viscoplasticity and time-dependent plasticity. Wang et al. [12] proposed a unified framework for the viscoplastic behaviour of SnAgCu solder materials which are referred to the Anand constitutive equations. The constitutive equation for Anand's model is:

$$\dot{\varepsilon}_p = A \exp^{\left(-\frac{Q}{RT}\right)} \left(\sinh\left(\xi \frac{\sigma}{s} \right) \right)^{\frac{1}{m}} \tag{3}$$

where $\dot{\varepsilon}_p$ is the inelastic strain rate, A the pre-exponential factor, Q the activation energy, R the universal gas constant, T the absolute temperature, σ the tensile stress, ξ the stress multiplier, m the strain rate sensitivity of the stress and s is the deformation resistance. In addition, $\dot{s}$ is an internal state variable whose evolution is described by:

$$\dot{s} = \left\{ h_0 \left| 1 - \frac{s}{s^*} \right|^a \operatorname{sign}\left(1 - \frac{s}{s^*} \right) \right\} \dot{\varepsilon}_p \tag{4}$$

where h_0 is the hardening or softening constant, a the hardening or softening strain rate sensitivity and s^* is the saturation value of associated with a given temperature and strain rate pair and is described by:

$$s^* = \hat{s} \left[\frac{\dot{\varepsilon}_p}{A} \exp^{\left(\frac{Q}{RT}\right)} \right]^n \tag{5}$$

where $\hat{s}$ is a coefficient for saturation and n is the strain rate sensitivity for s^*. The Anand's model is supported by the ANSYS code. The material parameters of Anand's model for the SA305 solder are obtained from experimental results and by the separated elasto-plasto-creep constitutive relations. These material parameters are shown in Table II as given by many authors.

Table I: Material properties.

description	value
FR4 CTE $\alpha_x = \alpha_y$ (ppm/K)	16
FR4 FR4 CTE α_z (ppm/K)	84
SAC Solder Young Modulus (GPa)	38.7
SAC CTE (ppm/K)	21
Liquidus temp. Of SAC305	218

Table II: Material parameters of Anand's model of SAC305 solder [10].

parameter	value
A (s^{-1})	5.87×10^6
Q/R (K)	7460
ξ	2
m	0.0942
$\hat{s}$ (MPa)	58.3
h_0 (MPa)	9350
a	1.5
s_0 (MPa)	45.9
n	0.015

The accelerated thermal profile testing recommended from JEDEC accelerated thermal cycling standards [13] is applied as the thermal loading in the finite Element analysis. These harsh temperatures vary between -40 C° and 125 C° with 30 min dwell at the peak and lowest temperature, the ramp rate is 15 C° /min are applied. Devices for automotive applications are typically tested within this temperature range. Figure 3 shows the cyclic temperature profile. Furthermore, this thermal cycling range can be suitable for testing microelectronics applications under the hood of automotive [10]. In the FEM calculus the material solders were assumed as stress free at the liquidus temperature. The first step consists to simulate the process of reflow soldering to take into account the initial stress. This process consists to apply the temperature profile going from the melting temperature of solder joints to room temperature of 25 C° in 150 seconds. In a second step, five thermal cycles between -40 C° and 125 C° are applied as loading condition in the finite element model.

2.3 Fatigue failure model

Most failures in electronic packaging are due to solder joints fatigue caused by the thermomechanical damage mechanisms during the component operation life. Solder joint fatigue life prediction involves combining finite element simulations with a thermal fatigue model. The fatigue model is generally obtained by using experimental data and accelerated testing. This model is used to determine the number of cycles that a package can resist before fatigue failure.

Several models have been proposed to predict solder joint fatigue life and may be classified into five categories: stress-based, plastic strain-based, creep strain-based, energy-based, and damage-based. Generally the stress-based approach is used in the high cycle fatigue where it is applied to vibration, shock and stressed components. On the other hand in low cycle fatigue, plastic strain-based and the creep strain-based approaches are used in low cycle fatigue. The thermal fatigue-induced strains from CTE mismatch fall under the second category. The plastic strain-induced fatigue is based on plastic deformation derived from the time-independent plastic material behaviour. Whereas the creep-strain-based approach is based on the creep deformation derived from the time dependent effects. The energy-based fatigue models are based on the calculation of the overall stress/strain hysteresis energy, where the damage-based fatigue models are based on computing the accumulated damage caused by crack propagation and fracture mechanics [14].

Since plastic strain is a dominant parameter that influences low-cycle fatigue, this study uses the Coffin-Manson type equation classified into the plastic strain-based approach. The inelastic strain is computed by nonlinear finite element simulation withthe unified viscoplasticity Anand's model, which combines plastic and creep deformation. The Coffin-Manson fatigue model is the most widely used approach, where the number of cycles to failure N_f depends on the plastic strain amplitude ϵ_p, generally given as the following equation:

$$\Delta\epsilon_p N_f^{\alpha} = \theta \tag{6}$$

Where $\Delta\epsilon_p$ is the plastic strain range, N_f is the fatigue life, α is the fatigue ductility exponent and θ is the fatigue ductility coefficient. A variety of strain life engineering data are available on most of the solders used in electronic packaging [15].

3. Reliability-Based Design Optimization

Deterministic structural optimization is frequently applied for effective cost reduction of engineering systems. In other words, it has been used as a decision tool in order to search the "best design" that minimize the structural cost and check performance requirements given by codes of practice. However, when the uncertainties related with geometrical dimensions, material properties loading, structural dimension and loading are not considered, the optimal solution may represents a lower level of reliability and high risk of failure. In fact, the deterministic design optimization reduces the remaining margins of the performance constraints to their lower bounds, thus the optimal design should be very sensitive to uncertainties. For this reason, the deterministic design optimization takes into account the uncertainties through the use of partial safety factors. These safety factors are assigned for some parameters (i.e. geometry, material properties, loads...) in order to allow a sufficient safety margin in the performance requirements. The General formulation of the deterministic design optimization is expressed as:

$$\begin{aligned} &\min_{\boldsymbol{d}} \quad : C(\boldsymbol{d}) \\ &\text{subject to:} \begin{cases} G(\boldsymbol{d}, \boldsymbol{x}_k, \boldsymbol{\gamma}) \leq 0 \\ h_j(\boldsymbol{d}) \leq 0 \qquad\qquad j = 1, \ldots, nc \end{cases} \end{aligned} \tag{7}$$

where $\boldsymbol{d}$ is the vector of design variables, $\boldsymbol{x}_k$ is the vector of characteristic values of load actions and material properties and other parameters that a are not considered as design variables, γ is the vector of safety factors, G is the performance limit state function and h_j are the feasibility constraints (e.g. upper and lower bounds of design variables). In this formulation, the partial safety factors introduced in the deterministic design are assumed to take account for uncertainties related to resistance and loading.

Nevertheless, these safety factors are defined by standard specifications for some engineering fields (i.e. Civil Engineering) or by engineering experience, where these factors are not linked to uncertainties and therefore the target safety level is not controlled in the deterministic optimization. As a matter of fact, the use of the safety factors in the deterministic design optimization may lead to poor design, as the iterative optimization procedure will search for the weakest region in the domain covered by the performance constraints. This weakest region often presents not only the lowest cost but also the lowest safety. The deterministic optimal design is pushed to the admissible domain boundaries, leaving very little space for safety margins of performance requirements [6].

The rational approach consists in finding the best compromise between cost reduction and reliability assurance, by taking the system uncertainties into consideration; where the probability theory provides an appropriate approach to take account for uncertainties [16]. The Reliability-Based Design Optimization (RBDO) allows us to reach effectively balanced cost-safety configurations and ensures economical and safe design. A practical formulation of the RBDO aims at minimizing the objective function under probabilistic constraints.

$$\begin{aligned} &\min_{\boldsymbol{d}} \quad : C_T = C(\boldsymbol{d}) \\ &\text{Subject to:} \begin{cases} \Pr\left[G(\boldsymbol{d},\boldsymbol{X}) \leq 0\right] \leq P_f^T \\ h_j(\boldsymbol{d}) \leq 0 & \quad j = 1, \dots, nc \end{cases} \end{aligned} \tag{8}$$

where C is the objective function (initial cost, structural volume,…,etc), defined in terms of design variables $\boldsymbol{d}$, P_f is the probability of failure and P_f^T is the admissible failure probability. The limit state function G is given in terms of design variables $\boldsymbol{d}$ and random variables $\boldsymbol{X}$. In the above model, the probabilistic constraints define the feasible domain, such as the failure probability P_f corresponding to the limit state G is kept lower than the allowable probability P_f^T. This failure probability is given by the integral:

$$P[G(\boldsymbol{d},\boldsymbol{X}) \leq 0] = \int \dots \int_{G(\boldsymbol{d},\boldsymbol{X}) \leq 0} f_{\boldsymbol{X}}(\boldsymbol{x})\, d\boldsymbol{x} \tag{9}$$

where $f_{\boldsymbol{X}}(\boldsymbol{x})$ is the joint probability density function of the random variables $\boldsymbol{X}$. The evaluation of this integral can be done either by Monte Carlo simulation techniques or by approximation methods, such as First Order Reliability Method (FORM). Monte Carlo simulations allow estimating the probability of failure for any general problem of the limit state function G (linear, nonlinear, continuous, discrete,…).In the context of the RBDO, Monte Carlo simulation techniques are avoided when the gradient-based optimization algorithms are used because the numerical noise due to the random sampling leads to the erroneous estimation of the response gradient and consequently induces the non-convergence of the optimization procedure. Moreover, Monte Carlo simulations are very high computational time, especially for realistic structures with low failure probability and when the mechanical model needs a numerical estimation such as finite element method. Approximation methods are usually preferred because of their numerical efficiency and accurate evaluation of gradients.

Another formulation of the RBDO can be done, particularly when the problem consists at searching the optimal robust solution that it is less sensitive to system uncertainties. In this sense, the reliability is maximized by minimizing the probability of failure under cost and structural constraints:

$$\begin{aligned} \min_{\boldsymbol{d}} \quad & : P_f = \Pr\left[G(\boldsymbol{d}, \boldsymbol{X}) \leq 0\right] \\ \text{Subject to:} & \begin{cases} C(\boldsymbol{d}) \leq C^t \\ h_j(\boldsymbol{d}) \leq 0 \qquad j = 1, \dots, nc \end{cases} \end{aligned} \tag{10}$$

Where, C^t is a target structural cost. This formulation aims at searching the optimal design solution by a better redistribution of the material within the structure by taking into account the effects of uncertainties and fluctuations that the reliability is maximized (or the probability of failure is minimized).

3.1 Reliability Analysis

The Approximation methods such as First Order Reliability Method (FORM) are usually used in the reliability analysis. The FORM method consists in approximating the limit state G_i by hyperplane at the most probable failure point (MPFP) [16]. The procedure of searching for the MPFP is formulated as an optimization problem under inequality constraint in the normalized space of the random variables:

$$\begin{aligned} \min_{u} \quad & \|u\| \\ Subject\ to & : G(u) \leq 0 \end{aligned} \tag{11}$$

where the random variables $\boldsymbol{X}$ are transformed into uncorrelated normalized random variables u. The solution u^{MPFP} is called the most probable failure point (MPFP), the reliability index β_i is equal to the distance between the origin and the MPFP. When FORM is used, the failure probability is approximated by $P_f = \Phi(-\beta)$, where $\Phi(\cdot)$ is the standard normal distribution. In this case, the probability constraints $\Pr\left[G(\boldsymbol{d}, \boldsymbol{X}) \leq 0\right] \leq P_f^T$ can be replaced by the reliability index constraints: $\beta(\boldsymbol{d}, \boldsymbol{X}) \geq \beta^T$; where $\beta(\boldsymbol{d}, \boldsymbol{X})$ and β^T are respectively the reliability index and the target index, relative to the limit state.

4. Adapted RBDO method with kriging approximation

From the numerical point of view, solving the RBDO problems is a heavy task because of the nested nonlinear procedures: optimization procedure, reliability analysis and numerical simulation as finite element method. The coupling between the optimization and reliability problems is a complex task and leads to a very high computational cost. This situation becomes impracticable for complex structures when finite element analysis considering nonlinear material behaviour and fatigue life prediction analysis are involved. Several methods have been developed to overcome the computational effort and the numerical difficulties [7].

However, the major difficulty is due to the evaluation of the structural reliability, which is carried out by a particular optimization procedure. In the random variable space, the reliability analysis implies a large number of mechanical calls, wherein the design variable space, the search procedure modifies the structural configuration and hence requires the re-evaluation of the reliability level at each iteration. For this reason, the solution of the RBDO problem requires very large computation resources that seriously reduce the applicability of this approach.

Kriging-model-based RBDO is an effective method to overcome this difficulty, where metamodeling techniques have been wildly used in engineering design optimization that the structural analysis involves expensive computational cost. In this work the kriging approximation is used to provide a surrogate model of the original thermomechanical finite element simulation of the solder joint. Besides, the kriging approximation is chosen in order to surrogate the performance functions because it develops an efficient accurate global approximation and allows one to quantify the approximation error. Thus, new sampling points are added to improve the prediction of the kriging approximation.

4.1 Kriging metamodel

The main advantage of the kriging approximation is that it gives an accurate global approximation while controlling the computational cost and accuracy. The construction of a kriging approximation model is based on data from a computer experiment. The kriging model then replaces the original computer model. The kriging approximation assumes a that the implicit function of the mechanical model is considered as the realization of a stochastic field $G(\boldsymbol{s})$. The first step is to define the stochasticfield parameters according to the computer design of experiments. The model for $G(\boldsymbol{s})$ is given as:

$$G(\boldsymbol{s}) = F(\boldsymbol{s}, \alpha) + z(\boldsymbol{s}) \tag{12}$$

where, $\boldsymbol{s} = \{\boldsymbol{d}, \boldsymbol{x}\}$ is a sample point that contains the design variables $\boldsymbol{d}$ and a realization of the random variables $\boldsymbol{x}$, $F(\boldsymbol{s})$ is the deterministic part which gives an approximation ofthe mean response. It shows the trend of the approximation and is represented by a regression model written as:

$$F(\boldsymbol{s}, \alpha) = \alpha_1 f_1(\boldsymbol{s}) + \alpha_2 f_2(\boldsymbol{s}) + \cdots + \alpha_p f_p(\boldsymbol{s}) \tag{13}$$

where, $\boldsymbol{f}(\boldsymbol{s}) = \{f_1(\boldsymbol{s}), \dots, f_p(\boldsymbol{s})\}$ is the vector of the basis functions and $\alpha = \{\alpha_1, \dots, \alpha_p\}$ the vector of regression parameters. $z(\boldsymbol{s})$ is a stationary Gaussian process with zero mean and covariance between two points of space $\boldsymbol{s}_l$ and $\boldsymbol{s}_2$ defined by:

$$COV((z(\boldsymbol{s}_1), z(\boldsymbol{s}_2)) = \sigma_z^2 R_\theta(\boldsymbol{s}_1, \boldsymbol{s}_2) \tag{14}$$

where, σ_z^2 is the process variance and R_θ the correlation model with parameters θ. Several models exist to define the correlation function. Here the anisotropic Gaussian model is used.

$$R_\theta((\boldsymbol{s}_1, \boldsymbol{s}_2) = \prod_{i=1}^{n} \exp\,[-\theta_i(\boldsymbol{s}_1 - \boldsymbol{s}_2)^2] \tag{15}$$

For a set S of m design of experiments $S = [s_1, s_2, \dots, s_m]$ where s_i $s_i \in R^{n_d + n_r}$ is the i^{th} sample point, where n_d is the number of design variables **d** and n_r is the number of random variables **X**. **G** is the vector of limit state function response for all the set of sample points, as:

$$\mathbf{G} = \left[G(s_1), G(s_2), \dots, G(s_m)\right] \tag{16}$$

where $G(s_i) = G(\mathbf{d}, \mathbf{x}) \in$ R. A is the expanded m×p design matrix with $A_{i,j} = f_j(s_i)$, as:

$$\mathbf{A} = \left[f(s_1), f(s_2), \dots, f(s_m)\right] \tag{17}$$

The matrix **R** of stochastic-process correlation between the design experiments is:

$$R_{i,j} = R_\theta\left(s_i, s_j\right) \tag{18}$$

Now, the parameters of the kriging approximation (θ,α,σ_z^2) should be determined. The Maximum Likelihood Estimation (MLE) technique is used as detailed by Lophaven et al. [17]. The likelihood of the data is maximized with respect to the parameters (θ,α,σ_z^2). The parameters α and σ_z^2 can be derived analytically using the Karush-Kuhn-Tucker (KKT) necessary optimality conditions (called also the first order optimality conditions), where it depends on the auto-covariance parameters θ. Thus, these parameters are estimated by solving the following algebraic equation:

$$\begin{aligned} \alpha^\star &= (\boldsymbol{A}^T\boldsymbol{R}^{-1}\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{R}^{-1}\boldsymbol{G} \\ \sigma_z^2 &= \frac{1}{m}(\boldsymbol{G} - \boldsymbol{A}\alpha^\star)^T\boldsymbol{R}^{-1}(\boldsymbol{G} - \boldsymbol{A}\alpha^\star) \end{aligned} \tag{19}$$

The best unbiased prediction of the response is:

$$\tilde{G}(\boldsymbol{s}) = \boldsymbol{f}(\boldsymbol{s}).\,\alpha^\star + r^T(\boldsymbol{s})\boldsymbol{R}^{-1}(\boldsymbol{G} - \boldsymbol{A}\alpha^\star) \tag{20}$$

DACE toolbox [17] is used to construct the kriging metamodel. A quadratic regression model and the anisotropic Gaussian model for the correlation function are selected. An anisotropic correlation function is preferred in reliability analysis studies [18]. The Latin hypercube sampling is used as a strategy for generating random sample points ensuring that all portions of the vector space is represented. For each random sample point the mechanical model is performed in order to compute the plastic strain. All these random sample points and their response quantities represent the design of experiments to be used in the building of the kriging metamodel. The mean square error (MSE) of Kriging approximation is equal to zero at the training point. However, at the testing points which are away from these training points, the MSEs increase highly.

4.1 Proposed RBDO approach with kriging approximation and adaptive sampling

In the reliability analysis, the local region in the vicinity of the most Probable Failure Point (MPFP) should be fitted accurately. The limit state constraint boundaries are more critical than the other regions. Lee et al. [9] have been proposed an efficient method called the Constraint Boundary Sampling (CBS) method that approximates accurately the limit state constraint boundaries over the design region. However, in the RBDO optimization process the local region in the neighborhood of the current design point should be also accurate than other regions. Chen et al. [19] propose to improve the accuracy of the kriging model on the limit state constraint boundaries which are in the relatively small region around the current design point, rather than making the whole limit state boundaries within the design region being accurate. This technique is combined with the CBS method to improve the prediction of the kriging approximation for the probabilistic constraint functions within the critical design region.

The prediction of $G(\boldsymbol{s})$ approaches the normal distribution with mean $\tilde{G}(\boldsymbol{s})$ and standard deviation $\sqrt{MSE(\boldsymbol{s})}$, where $\sqrt{MSE(\boldsymbol{s})}$ is the mean square error of the kriging prediction. If the failure region is defined as $G(\boldsymbol{s}) \leq 0$, then the probability of the kriging prediction satisfies the constraint $G(\boldsymbol{s}) > 0$ is given by:

$$p(\boldsymbol{s}) = \Phi\left(\frac{\tilde{G}(\boldsymbol{s})}{\sqrt{MSE(\boldsymbol{s})}}\right) \tag{21}$$

The probability density function $\phi\big(\tilde{G}(\boldsymbol{s})/\sqrt{MSE(\boldsymbol{s})}\big)$ can be used to measure the closeness of the kirging prediction $\tilde{G}(\boldsymbol{s})$ to the limit state function $G(\boldsymbol{s}) = 0$ [19]. The CBS Criterion is defined as:

$$CBS = \begin{cases} \phi\left(\frac{\tilde{G}(s)}{\sqrt{MSE(s)}}\right).D & \text{if } \tilde{G}(s) > 0 \\ \quad 0 \quad \text{otherwise} \end{cases} \tag{22}$$

where D is the minimal distance from the current sample point $\boldsymbol{s}$ to the other sample points. If the region does not contain any constraint boundary, then the MSE multiplied by D can be used as sampling criterion [19].

These two techniques are used in the proposed method in order to improve the prediction of the kriging approximation in the RBDO approach. The first step of proposed approach is based on the construction of the kriging approximation on the basis of a set design experiments generated by the Latin Hypercube Sampling method. These design experiments correspond to the design and the random variables. The first step consists to perform the RBDO problem formulated in Eq. 10 or 13, where the probability of failure is estimated by using FORM method (Eq. 14). The third step consists to estimate the CBS criterion of the optimal design point obtained at the previous cycle of the RBDO problem. If the criterion is not satisfied, then a new set of sample points generated by the HLS method in the neighborhood of the optimal point are added to the existing design of experiments to update and improve the kriging metamodel. A new RBDO problem is solved by using the updated kriging approximation. This iterative scheme with updating the kriging metamodel and RBDO cycles is repeated until convergence. The proposed approach is illustrated by the flowchart in Figure 5.

5. Numerical Example

The optimal configuration of the solder joint in figure 6 is searched by minimizing the probability of failure by using the RBDO formulation expressed in Eq. 10. The design variables are the thickness of the solder e_s, the length of the right side of the solder $L_{s,r}$ and the ratio α giving the length of the left side of the solder by $L_{s,l} = \alpha L_s$ where L_s is the length of the copper gull-wing lead fixed by the packaging technology [20]. In this study the value of L_s is fixed to 0.7 mm. All the design variables are the means of the random variables assumed statistically independent and have a normal distribution with coefficient of variation of 3%. The peak and the lowest temperatures $\{T_p, T_l\}$ of the cyclic temperature profile are considered as random variables with mean values respectively 125 C° and -40 C° with coefficient of variation of 5% and assumed to follow the normal probability distribution.

The RBDO solder joint problem consists at searching the optimal value of e_s,$L_{s,r}$ and $L_{s,l}$ that minimize the probability of failure and ensure the design feasibility. The probability of failure is written in terms of the plastic strain ε_p that should be lower than the allowed plastic strain ϵ_p^t. This allowed plastic deformation is considered as a random variable with mean of 0.01 and coefficient of variation of 10% and assumed to follow a normal probability distribution. The RBDO solder joint problem is formulated as:

$$\text{find } \boldsymbol{d} = \{e_s, \alpha, L_{s,r}\} \text{that:} \min_{\boldsymbol{d}} P_f = \Pr\left[\epsilon_p(\boldsymbol{d}, \boldsymbol{X}) \geq \epsilon_p^t\right]$$
$$\text{Subject to:} \begin{cases} Area(\boldsymbol{d}) \leq 0.4414\, mm^2 \\ 0.1\, mm \leq e_s \leq 0.3\, mm \\ 0.2 \leq \alpha \leq 0.9\ (3) \\ 0.2\, mm \leq L_{s,r} \leq 0.9\, mm \end{cases} \tag{23}$$

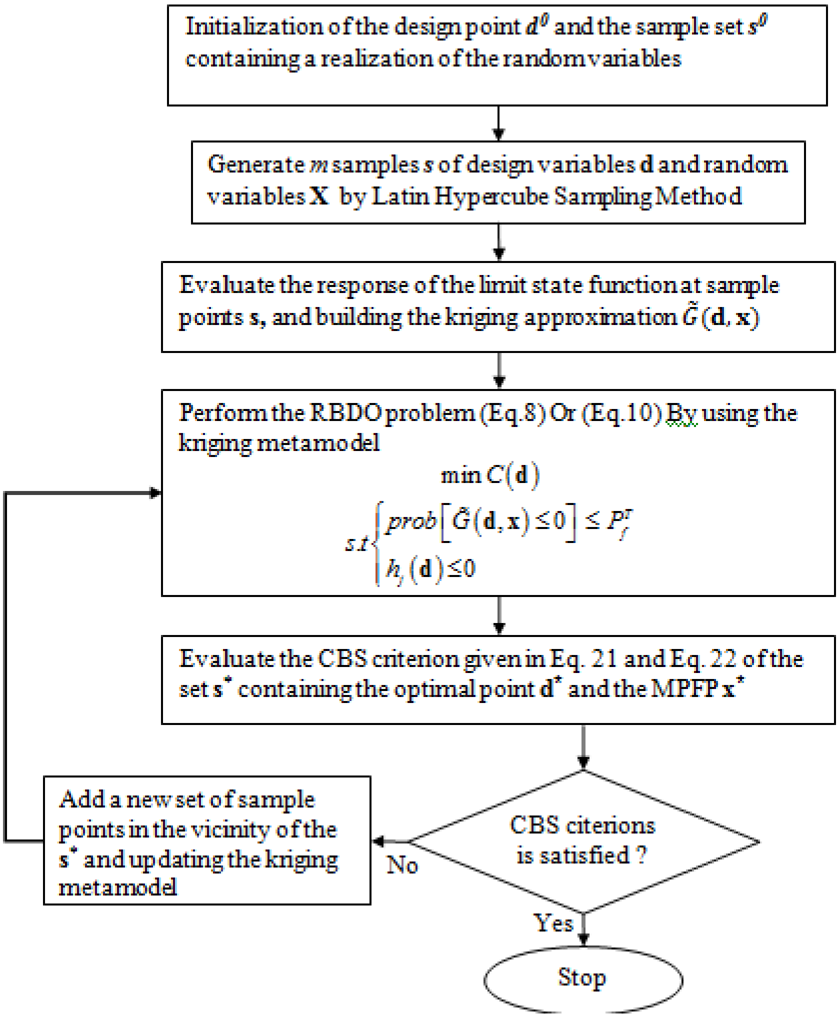

Figure 5. Flowchart of the Proposed RBDO approach with kriging approximation**.**

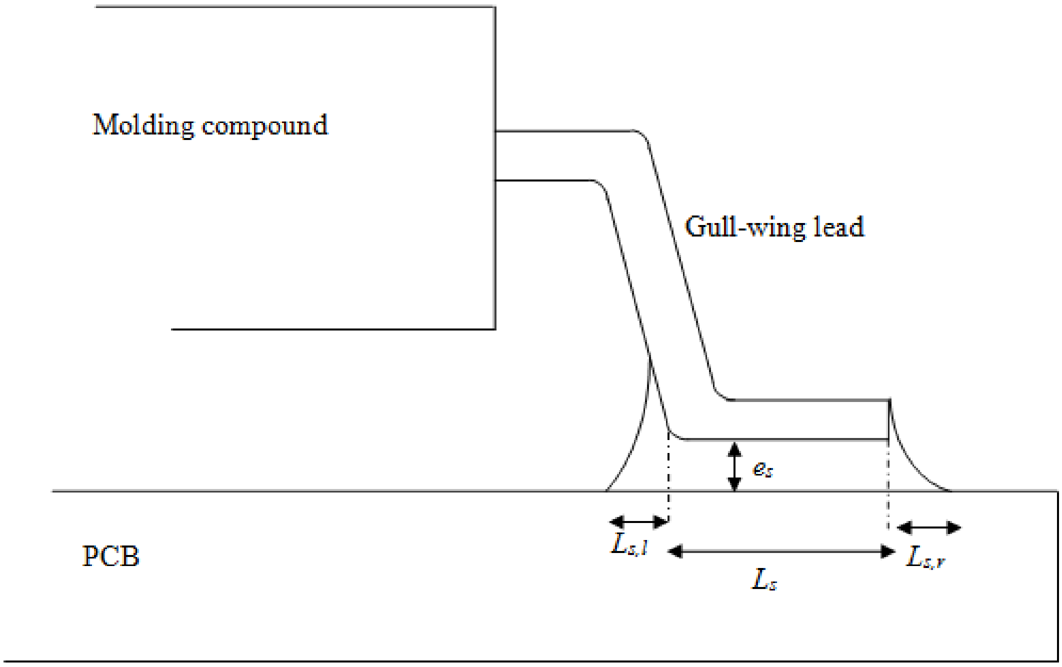

Figure 6. Design variables of the solder joint optimization problem.

As is shown in the flowchart of the proposed approach in Figure 5 the first step consists to build the kriging approximation of the accumulated plastic strain per cycle with respect of the design variables $\{e_s, \alpha, L_{s,r}\}$ and the random variables $\{T_p, T_l\}$. Figure 7 shows the representation of the kriging approximation where the approximated plastic strain is plotted regarding the left length given by α, the right length $L_{s,r}$ and by fixing the value of the solder thickness respectively in 0.1mm, 0.15mm, 0.2mm, 0.25mm or 0.3mm. The peak and the lowest temperatures $\{T_p, T_l\}$ are being fixed to their mean value. Figure 8 shows the Mean squared Error of this approximation. The relative error does not exceed 3.5%.

The optimal design of the solder joint that verify the maximum reliability is estimated with the RBDO approach formulated in Eq.10. Figure 9 show the history of the reliability index during iterations, as the initial reliability index of the initial design corresponds to 1.69 corresponding to the probability of failure of 0.045. The reliability index of the optimal configuration of the solder joint is maximized until 2.24 corresponding to the probability of failure of 0.012. The optimal design estimated by the proposed RBDO method improves the reliability index, where the probability of failure is then reduced. Table III shows the mean values of the optimal configuration of the solder joint. The probability of failure of the optimal solution is then estimated by Monte Carlo simulations by using the last updated kriging approximation. The results given in this table show the correctness of the FORM approximation results to estimate the probability of failure from the reliability index.

Figure 10 indicates the optimal configuration of the solder joint estimated from the RBDO approach. Figure 11 shows the stress magnitude as a function of strain magnitude, also known as hysteresis loop of the initial design and the optimal design of the solder joint. The area under the hysteresis loop represents the strain energy that is stored in the solder joints. High area value leads to a reduction of the fatigue strength. Figure 11 indicates that the area of the hysteresis loop of the initial design of the solder joint is larger than the hysteresis loop area of the optimal configuration searched by the RBDO approach. This design of solder joint reduces significantly the plastic strain of the solder joint, thereby improves the fatigue life.

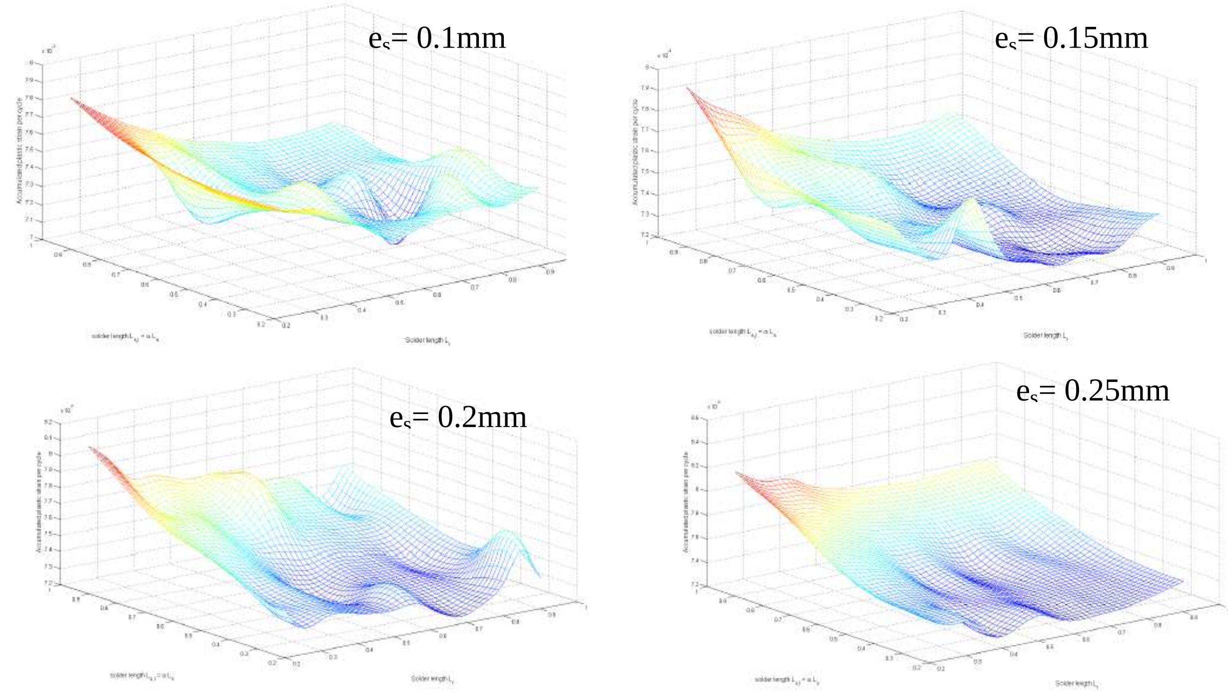

Figure 7. Approximation of the accumulated plastic strain per cycle regarding the solder lengths.

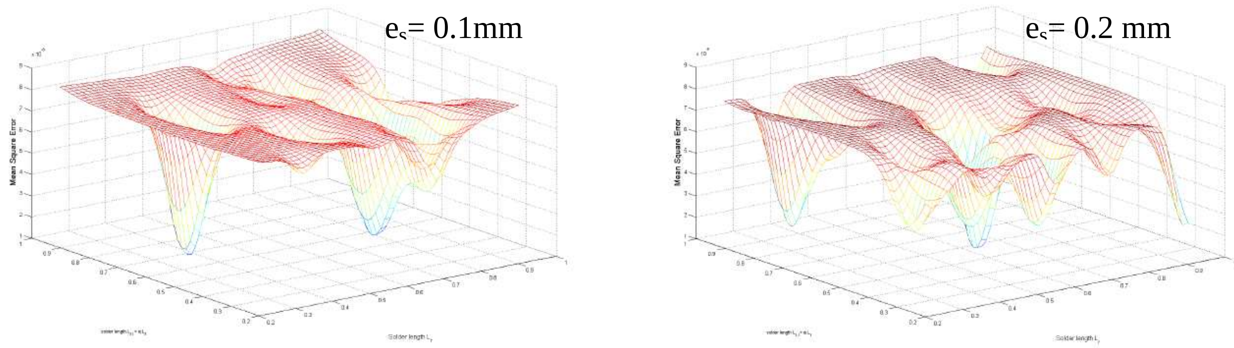

Figure 8. Mean Square Error of the plastic strain approximation in terms of the solder lengths

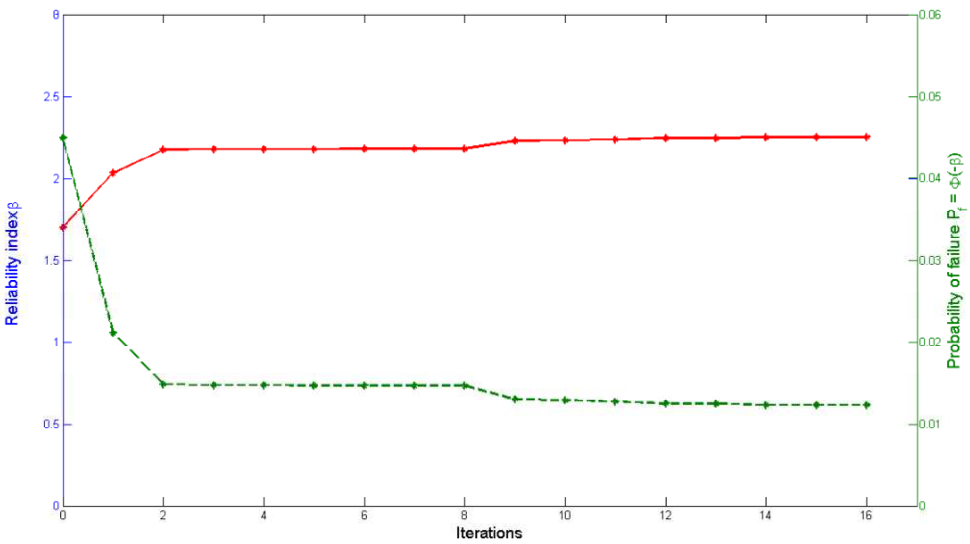

Figure 9. Reliability index and probability of failure history.

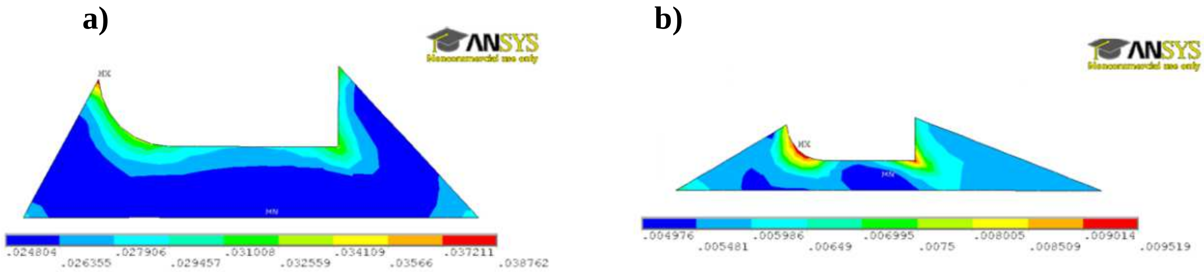

Figure 10. a) Initial solder joint design b) Optimal RBDO solder joint design.

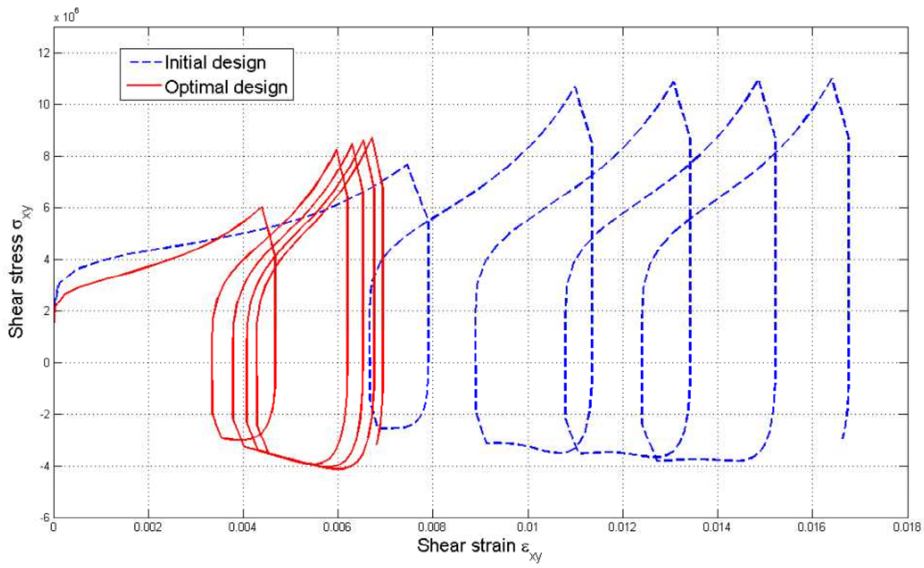

Figure 11. Hysteresis loop of the initial design and the RBDO optimal design.

Table III: Numerical results for optimal design of the solder joint.

Times 10 pt bold	**Initial design**	**Optimal design**
Solder thickness e_s (mm)	0.15	0.1284
Left length of the solder $L_{s,r}$ (mm)	0.3	0.7143
Right length of the solder $L_{s,l}$ (mm)	0.63	0,3305
Reliability index β	1.6966	2.2484
Probability of failure with FROM	0.0449	0.0123
Probability of failure with MCS (100000 samples)	0.0448	0.0150
Probability of failure interval with 95% from MCS	[0.0435 ; 0.0460]	[0.0142 ; 0.0157]

6. Conclusions

In this study the Reliability-based Design Optimization approach is used to improve the fatigue strength of the design of the solder joint of a mechatronic packaging. Since plastic strain is a dominant parameter that influences low-cycle fatigue, the probability of failure expressed in terms of the plastic strain per cycle is minimized in order to find the best geometrical configuration of the solder joint. However, the plastic strain is estimated form a nonlinear thermomechanical finite element analysis, where it is time consuming. A kriging approximation is used to achieve the RBDO problem. A proposed approach of the RBDO formulated by searching the optimal design that minimizes the probability of failure is combined to the kriging metamodel and adaptive sampling. The effectiveness of the proposed method is demonstrated by performing the optimal-reliable design of the solder joint.

The uncertainties related to the variability of the geometry and the thermal loading fluctuations have a strong effect on the lifetime of the electronic package. So far, the RBDO approach is used to improve design robustness and to reduce the impact of parameter uncertainties on the fatigue life.

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References

[1] Lu, H., Bailey, C. and Yin, C. (2009) Design for reliability of power electronics modules. *Microelectronics Reliability*, vol. 49, pp. 1250-1255.

[2] Wymysłowski, A., Zhang, G.Q., Driel, W.D. and Ernst, L.J. (2007) Virtual Thermo-Mechanical Prototyping of Microelectronics and Microsystems. In: Suhir, E., Lee, Y.C. and Wong, C.P (eds) *Micro- and Opto-Electronic Materials and Structures: Physics, Mechanics, Design, Reliability, Packaging*, Springer US, pp. 205-A266.

[3] Mao, C.Y. and Chen, R.S. (2008) Packaging parameter analysis and optimization design on solder joint reliability for twin die stacked packages by variance in strain energy density (SED) of each solder joint. *Microelectronics Reliability*, vol. 48, pp.119–131.
[4] Xu, L., Reinikainen, T., Ren, W., Wang, B.P., Han, Z. and Agonafer. D. (2004) A simulation-based multi-objective design optimization of electronic packages under thermal cycling and bending. *Microelectronics Reliability*, vol. 44(12), pp.1977-1983. 2004.
[5] Wang B.P., Xue, Z., Han, Z., Xu, L. and Reinikainen, T. (2005) A novel response surface method for design optimization of electronic packages, In: *Proceedings* (EuroSimE) *of the 6th International Conference on Thermal, Mechanical and Multi-Physics Simulation and Experiments in Micro-Electronics and Micro-Systems, Berlin, Germany, April* 2005, pp. 175-181.
[6] Chateauneuf, A. (2008) Principles of Reliability-Based Design Optimization, In: Tsompanakis, Y., Lagaros, N.D. and Papadrakakis, M (eds.) *Structural design optimization considering uncertainties.* Taylor & Francis, chap 1, 2009
[7] Aoues, Y. and Chateauneuf, A. (2010) Benchmark study of numerical methods for reliability-based design optimization. *Structural & Multidisciplinary optimization*, vol. 41(2), pp. 277–294, 2010.
[8] Dubourg, V., Sudret, B. and Bourinet, J.M. (2011) Reliability-based design optimization using Kriging surrogates and subset simulation. *Structural &Multidisciplinary optimization*, vol. 44(5), pp. 673–690.
[9] Lee, T.H. and Jung, J.J. (2008) A sampling technique enhancing accuracy and efficiency of metamodel-based RBDO: constraint boundary sampling. *Computers Structures*, vol. 86(13-14), pp. 1463–1476.
[10] Otiaba, K.C., Bhatti, R.S., Ekere, N.N., Mallik, S. and Ekpu, M. (2013) Finite element analysis of the effect of silver content for Sngu alloy compositions on thermal cycling reliability of solder die attach. *Engineering Failure Analysis*, vol. 28, pp. 192–207.
[11] Syed, A. (2007) Accumulated Creep Strain and Energy Density Based Thermal Fatigue Life Pre-dictionModels for SnAgCu Solder Joints. In *54th Electronic Components and Technology Conference Proceedings, Las Vegas, Nevada*, USA, pp. 737–746.
[12] Wang, G.Z., Cheng, Z.N., Becker, N. and Wilde, J. (2001) Applying Anand Model to Represent the Viscoplastic Deformation Behavior of Solder Alloys. *Journal of Electronic Packaging*, vol.123, pp. 247–253.
[13] JEDEC Standard, Temperature Cycling. *JEDEC SOLID STATE TECHNOLOGY ASSO-CIATION*, JESD22-A104D, 2009.
[14] Lee, W.W., Nguyen, L.T. and Selvaduray, G.S. (2000) Solder joint fatigue models: review and applicability to chip scale packages. *Microelectronics Reliability*, vol. 40, pp. 231–244.
[15] Shang, J.K., Zeng, Q.L., Zhang, L. and Zhu, Q.S. (2007) Mechanical fatigue of Sn-rich Pb-free solder alloys. In: *Lead-Free Electronic Solders*, Springer US, pp. 211–227.
[16] Ditlevsen, O. and Madsen, H.O. (1996) *Structural reliability method*. New York : John Wiley and Sons, 1996.

[17] Lophaven, S.N., Nielsen, H.B. and Sondergaard, J. (2002) Aspects of the Matlab Toolbox DACE. *Informatics and Mathematical Modelling*, DTU, Report IMM-REP-2002-13.
[18] Kaymaz, I. (2005) Application of Kriging method to structural reliability problems. *Structural Safety*, vol. 27(2), pp. 133–151.
[19] Chen, Z., Qiu, H., Gao, L. and Li, X. (2014) A local adaptive sampling method for reliability-based design optimization using kriging model. *Structural & Multidisciplinary optimization*, vol. 49, pp. 401–416.
[20] Renesas Electronics Corporation, JEITA Package Code P-QFP256-28x40-0.50 RENE-SAS Code PRQP0256KB-A. *Technical documentation of Renesas Electronics Corporation*, http://www.renesas.com.